Linear algebra: In this subject, we will discuss the system of linear equations.

Consistent: If a system has a solution then it is said to be consistent.

Solvable, has a solution,

Inconsistent: (Not consistent) A system which does have a solution is term as inconsistent.

6x+5y=6 Two relations joined with the sign of equality.

Linear equation:

X+2y=3

4x+5y=6

Ax=b

Elimination method and cramer rule

More than three or four system of linear equations

Guass elimination method

2x+3y=1

10x+9y=11

Ax=b

Augumented matrix:

R2+(-5)R1

2x+3y=1

-6y=6 implies that y=-1

x=2

x+y=4

2x-2y=4

X=3,y=1

Consistent (has a solution)

Matrix: A rectangular array of elements enclosed by vertical lines.

its order is 3by2. Square matrix

Diagonal matrix:

Scalar matrix:

Symmetrix matrix: At=A

off diagonal entries are mirror image of each other.

Skew symmetric matrix:

At=-A

Permutation matrix: this matrix will help us to change the rows and columns of given matrix.

Left side sy multiply to row exchange

Right side sy multiply kry gy to columns changes

Multiplication is conformable when the number of columns of first matrix is equal to the number of rows in the second matrix.

A is of order 3by3 and the column vector is of order 3by1. So multiplication is of order 3by1 and it requires 9 multiplication during its computation.

A is of order mbyn and the order of B is nbyp.

So the order of AB is mnp.

Linear combination:

Aj, Ao and Ah.

Ah=c(Aj)+d(Ao)

c(Aj)+d(Ao)+e(Ah)=0

c,d and e all have values equal to zero then Aj, Ao and Ah are linearly independent.

If any on of the c,d and e are non-zero then they are said to be linearly dependent.

U stands for upper triangular matrix and is the matrix whose all entries are zero below the main diagonal and non zeros entries on the diagonal and above the diagonal.

upper triangular matrix

A and want to change that matrix in the upper triangular matrix then we have to apply elementary row operations.

Elementary matrices will be obtained from identity matrix by making some changes in it according to the applied row operations.

U

In general matrix multiplication is not commutative that AB isnotequalto BA

Diagonal matrix: A matrix whose all entries are zero below the main diagonal and then entries on the diagonal and above the diagonal are nonzero.

Triangular matrix: Diagonal matrix, scalar matrix, lower triangular matrix and upper triangular matrix. Their determinant will be equal the product of diagonal entries. It means if one of the diagonal entry is zero then that matrices becomes singular matrix.

A=LU

Lux=b

Put ux=c then

Lc=b

If you want to write a permutation matrix then you can write an identity matrix and then changes its rows or column to get permutation matrix.

If there is no need to exchange the rows or column then we can choose L as an identity matrix.

When you are going to exchange the rows then you are going to multiply with permutation matrix from the left side and when you want to exchange the columns then you are going to multiply from the right hand side.

3+2+1=6=n(n+1)/2

n-1+n-2+n-3+….+1=(n-1)(n)/2

Inverse: Inverse is possible when its determinant is zero.

If a matrix has n pivots then its determinant will be non zero.

You can get pivot by converting the given matrix in upper triangular matrix by using elementary row operations.

If there is no non zero vector in the null space of A then that matrix is invertible mean non singular.

You know that the inverse of a square matrix having order 2 is the ration of adjoint of A over determinant of A. you know that you can take the adjoint of 2by2 matrix by taking the -ve sign of the third and fourth given entries and by changing the position of first and fourth one.

The determinant of Triangular matrix is equal to the product of diagonal entries.

The inverse of a permutation matrix is the transpose of that matrix.

Guass Jordan method. We will use this method to calculate the inverse of given matrix. For this, we have to append identity matrix on right hand side with the given matrix.

After we have to apply the elementary row operation in such a way that you can get identity matrix on the position of given matrix and the matrix which we get at the position of identity matrix is our required inverse of the given matrix.

(AB)-1=C-1 implies that B-1A-1=C-1 implies that A-1=BC-1

Control system, robotics, Digital signal processing, Machine learning and so many other fields.

If a system is singular then it may or may not have a solution. If it has a solution then it has infinite number of solutions.

If a system is non singular matrix then it has a unique solution.

A=LU

A=LDU, for symmetric matrix U becomes LT .

A=LDLT

Invertible mean non singular mean that its inverse is admissible.

(AB)-1=B-1A-1

M=ABC

M-1=(ABC)-1

=C-1B-1A-1

CM-1A=B-1

A matrix is said to be a singular if its determinant is zero.

When determinant can be zero.

When two rows or columns are identical.

When one complete row or column is zero

When one row or column is a multiple of other.

Singular is also termed as noninvertible matrix.

Remember that the matrix multiplication is not commutative.

It means that AB is not equal to BA.

A(BC)=(AB)C Associative law

Block Matrix: A matrix whose entries are itself a matrix.

Capital letters will be used for matrix notation and small alphabets will be used both for vector notation as well as for the entries of the given matrix.

MM-1=I

I

A non zero set which satisfies 8 conditions is termed as vector space.

Field means that is its dimension is one. Scaler we will take from field.

Whether the set of natural number is a field or not.

N={1,2,3,4,5,……}

W={0,1,2,3,….}

Set of real number and set of rational both are vector space.

Subspace: A subset of a vector space which is also a vector space is termed as subspace.

To show that the given set is a Subpace or not then we have to check the two conditions and that two conditions are

1. Sum of any two elements of the set must belong to that set that It must be closed w.r.t addition.
2. Scaler multiplication with any vector of the given must belong to that set.

These two conditions together are linear combination of elements of the set must belong to that set.

Set of natural number is a subspace or not of set of real number.

R3 its exponent is three so there will be three elements in each vector of R3.

X+2y+z=6

X+2y+z=0 (impossible means that these lines are parallel and they do not have a solution).

In linear algebra when I will call a vector it means that it is a column vector unless I stated about that vector.

Set of all nonsingular matrices of set of all two by two matrices. You have to check whether the set of all nonsingular matrices is a subspace or not.

singular, non-singular, symmetric and skew symmetric, diagonal, scalar, identity and so on.

X+y-2z=4

(1,1,-1) and (0,0,-2)

(1,2,-3)

Column space: The space generated by the columns of A and it is equal to Ax=b. This will be solvable when b can be written as a linear combination of columns of A.

Column space that is actually generated by the linearly independent columns of A.

A is of 3by2 order so the order of x must be 2by 1.

Ax=b is solvable when

r[A b]=r(A).

when b can be written as a linear combination of columns of A then we say that our system is solvable.

When b lies in the column space of A then we say that Ax=b has a solution.

x=xp+xn this is known as complete solution. It means that a complete solution is sum of particular and special solution and

Ax=A xp+Axn =b+ 0=b

xp= Particular solution

and

xn = special solution

when we have to reduce our given matrix in the form of reduce echelon form, the pivot column are those column whose pivot is non zero and whose pivot is zero is known as free column.

Rank: the number of nonzero rows in echelon or reduce echelon form is known as rank.

The number of pivot column (column having non zero pivot) is known as rank.

The number of linearly independent column is known as rank.

A3X4x4X1=0

It means that null space belong to R4.

A3X4x4X1=b

It means that column space belong to R3.

A31X14x14X1=b

A3X4

Rank is the number of nonzero rows in echelon or reduce echelon form or the number of linearly independent columns.

Linear Combination: v1,v2,v3 are three vectors. Their linear combination is c1v1+c2v2+c3v3

c1v1+c2v2+c3v3 =0

and all ci’s, i=1,2,3 are zero then the given vectors are said to be linearly independent otherwise linearly dependent.

A set of vectors is said to be spanning set of R3 then every element in R3 can be written as linear combination of elements of that set.

A={(1,2), (2,5),(3,7)} under this condition is set is linealry dependent and span of R2 .

A={(1,2), (2,5)} this I come to know that this set is linearly independent as well as a spanning set of R2 so it’s a basis of R2.

5=a+2b implies that a=5-2b implies that a=5-2(-3)=5+6=11

7=2a+5b

7=2(5-2b)+5b

7=10-4b+5b

7=10+b

b=-3

A={(1,2),(2,9)} redundant

Basis:

Is this set is a spanning set of R2 or not.

4=a+2b implies a=4-2b=4-2(-1/5)=4+2/5=22/5

7=2a+9b=2(4-2b)+9b=8-4b+9b

b=-1/5

R4

B={(1,2,3),(2,3,4),(3,5,7)}

Is this set a basis of R3

R10

Four fundamental subspaces:

Column space: Ax=b ,

, Ax=0, yA=0 , Atx=0

Aty=b

Column (row) space: The space generated by the columns (Rows) of A is termed as column (row) space of A. Some columns (rows) are linearly dependent and some are Linearly independent. The columns (rows) which are linearly independent they form the basis of column (row) space of A. There is no contribution of Linearly dependent column (rows) in the column (row) space of A because that linearly dependent columns (rows) can be generated from the support of Linearly independent columns (rows). The row is actually the column space of At.

Null space: Ax=0 in this case x belongs to null space of A. The number of free columns indicate that these number of columns lies in the null space of A.

Left null space: yA=0 why because the vector is going to multiply from left hand side so this is known as left null space.

Atx=0 this is the condition for left null space.

The vectors which lies in the Column space are always perpendicular to the vectors which lies in the left null space.

The vectors which lies in the row space are always perpendicular to the vectors which lies in the null space.

Ax=0

[]

If the matrix is square and non singular then you can calculate its inverse. But when the matrix is square but singular or even rectangular matrix then we can not calculate its inverse. Then we will calculate its left inverse or right inverse.

If A is invertible it means that

A-1A=AA-1=I

AX=I then we say that this X is its right inverse.

When the rank of given matrix is equal the row of the matrix then its right inverse will exist.

It means that the order of identity matrix is equal to rows of matrix.

A[At(AAt)-1]=I

[(AtA)-1At]A=I

Linear transformation:

f:x

f(1)=2

T: v

V and W are vector space so there elements can vector having 1, 2 or three and so on components.

This T is linear if it follows the following two conditions

1. T(v1+v2) =T(v1)+T(v2)
2. T(av)=aT(v) scalar multiplication

I can write these two properties jointly as

T(av1+bv2)=aT(v1)+bT(v2) , a,b, F stands for field and its dimension is 1. Actually field is just R1.

Polynomail: f(x)=a+bx

g(x)=a+bx+cx2

h(x)=a+bx+cx2+dx3

Basis:

Linearly independent

Span

Ax=b

Perpendicular: Here the alternative word for perpendicular is orthogonal.

Vectors means column vecotrs and vectors are said to be orthogonal when their inner product is zero. Mathematically, for given vectors a and b, we have atb=0.

<a,b>= atb=0 the inner product of a & b is zero means these vectors are orthogonal/perpendicular/normal.

Length or magnitude: here in linear algebra these terms are replaces with norm of vector.

x=(x1,x2,x3) implies that its length is =

Now I will tell about the relationship between norm and inner product.

x=(x1,x2,x3)

xtx= =.

Four fundamental subspaces:

Column space, Null space, Row space and left null space.

Column space will always be perpendicular to left null space.

Row space will always be perpendicular to null space.

A null vector is always linearly dependent.

a=weight, scaler

Remember that row space and null space are always perpendicular to each other. The null space is the orthogonal component of row space.

perpendicular to each other. The left null space is the orthogonal component of column space of A.

x+2y-z=0

x=-2y+z

(x,y,z)=(-2y+z,y,z)=y(-2,1,0)+z(1,0,1)

AB=0

B lies in the null space of A.

A lies in the left null space of B.

Basis: Linearly independent

Span

Four fundamental subspaces:

Column space

Null space

Row space

Left null space:

If there is no solution for the system of equations then we will calculate its least square solution.

Ax=b

AtAx=Atb normal equation

x=(AtA)-1Atb least square solution

Axcap=A(AtA)-1Atb this is known as projection of b onto column space of A.

P= A(AtA)-1At

P2=P and Pt=P

Orthogonal matrix:

A square matrix for which AtA=I=AAt

In general, we will use Q for orthogonal matrix.

Qt=Q-1

Permutation matrix: A matrix obtained by swapping the rows or columns of an identity matrix.

Remember that the inverse of a permutation matrix is always its transpose.

Orthogonal vector: If the dot product of two vectors is zero the that vectors are termed as orthogonal vectors.

Additionally, if the length of each vector is 1(unit) then that vectors are said to be orthonormal vectors.

Rotation matrix: ,

=

Atb=0, this mean that these vectors are orthogonal/perpendicular/normal.

Gram Schimdth Method:

This method is used to find out the orthonormal vectors.

a, b, are non orthogonal vectors.

Norm mean length and it is equal to aat

I have to made them as orthonormal vectors.

2by2.

3by3

I am going to write the vector in column wise fashion. You can also write it in row wise fashion.

If you are going to calculate the determinant of triangular (upper/Lower), scalar matrix, diagonal matrix or identity matric then you have to multiply the diagonal entries to get the value of determinant.

Determinant of A-1=1/detA.

Orthogonal matrix: AAt=I implies that At=A-1

A matrix whose inverse is equal to the transpose of given matrix is termed as orthogona.

If there is a skew symmetric matrix of odd order then its determinant will be zero.

this is a skew symmetric matrix of order 3 which is odd. So its determinant must be zero.

, its order is n and n is odd.

det(kt)=det(-1.k)

det(k)=(-1)ndet(k)

as we know that this n is odd and (-1)n=-1 because n is odd

det(k)=-det(k)

2det(k)=0

Det(k)=0

det(S-1AS)=det(S-1)det(A)det(S)=

det(S-1S)det(A)=det(I)det(A)=det(A)

Jacobean is mostly used in robotics.

Cartesian coordinates: x, y and z

Polar coordinates: , this is the relationship between polar and cartesian coordinates. r and

Cylindrical coordinates:

Spherical coordinates: and their relationship to cartesian coordinates is

Ax=b

Ax=x, eigen value and x= eigen vector

Ax-x=0

Ax-x=0

the vectors which lie in the null space of A-

Characteristic equation

The roots/solutions of characteristic equation are termed as eigen values of given matrix A.

Diagonalization of a matrix: TO find out the diagonalization of given matrix we have to follow the following steps.

We will calculate eigen values.

You have to calculate the corresponding eigen vectors to the calculated eigen values (A-)x=0

You have to construct a matrix S by writing the eigen vector in the column wise fashion.

S-1AS= its diagonal entries will be the eigen values of given matrix.

A= S S-1

A5= S S-1 S S-1 S S-1 S S-1 S S-1

=S5 S-1

When we get the different eigen values then we get the corresponding eigen vectors smoothly. Means to sat that sufficient eigen vectors are available to compute the diagonalization of given matrix.

When we get similar eigen values then may be we are not in a position to calculate the sufficient number of eigen vectors and when there is not sufficient number of eigen vectors then diagonalization of that matrix is not possible.

A matrix which is not able for diagonalization is termed as defective matrix.

Algebraic multiplicity and geometric multiplicity

Algebraic multiplicity: Number of times, a eigen value appear is called as it’s a.m.

Geometric multiplicity:

Diagonalization:

S-1AS=

dv/dt=w  
dw/dt=v